Geometry Final Exam Review

Chapter 6 (All sections)

- Ratio and proportion (geometric mean)
  - If *a* and *b* and $b \neq 0$, then the **ratio of a to b** is $\frac{a}{b}$
  - An equation that states that two ratios are equal, such as $\frac{a}{b} = \frac{c}{d}$, is called a

proportion. The numbers *b* and *c* are the **means** , and the numbers *a* and *d* are the **extremes.**

  - The **geometric mean** of two positive numbers *a* and *b* is the positive number *x* that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$

*Examples of **geometric mean**:*

2 and 12	$\sqrt{176}$	$\sqrt{98}$
$\sqrt{24}$	$\sqrt{16*11}$	$\sqrt{49*2}$
$\sqrt{4*6}$	$\sqrt{16}\sqrt{11}$	$\sqrt{49}\sqrt{2}$
$\sqrt{4}\sqrt{6}$	$4\sqrt{11}$	$7\sqrt{2}$
$2\sqrt{6}$		

- Use proportions to solve problems

KEY CONCEPT *For Your Notebook*

Additional Properties of Proportions

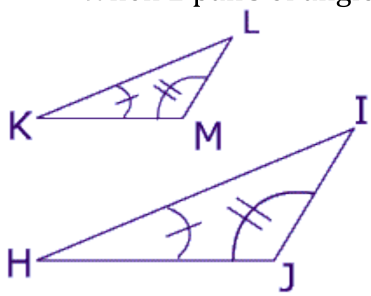
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. — If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.
3. If you interchange the means of a proportion, then you form another true proportion. — If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.
4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. — If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

- Similar polygons (scale factor)

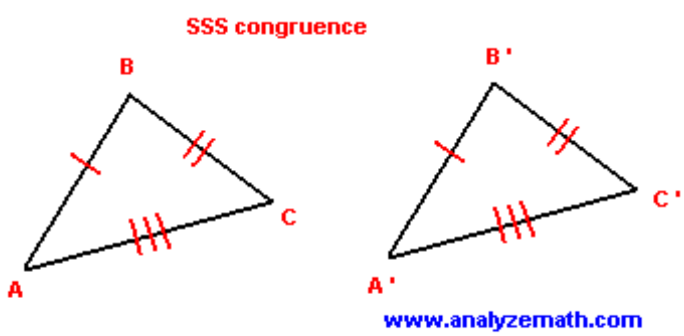
- A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how dimensions in the drawing are related to the actual dimensions of of the object.

- **Additional properties of proportions**
  - Reciprocal Property:
    - If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$
    - If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$

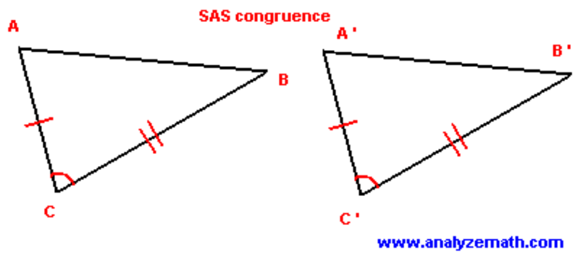
- Similar triangles (AA similarity postulate)
  - **AA Similarity Postulate:**
    When 2 pairs of angles are the same (congruent), then the triangles are similar.



- Proving triangles similar (SSS and SAS similarity theorems)
  - **SSS Similarity Theorem:**
    If the corresponding side lengths of the side lengths of two triangles are proportional, then the triangles are similar.



- **SAS Similarity Theorem:**
  If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



- Proportions and similar triangles
  - Scale factor= same as perimeter ratio
  - The area ratio is the scale factor squared.

Chapter 7 (All sections)

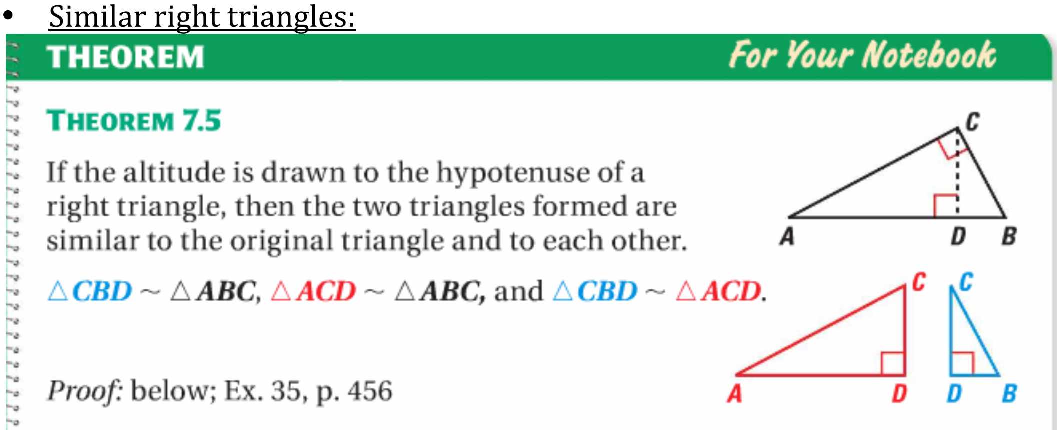
- Pythagorean Theorem :
  $a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem: If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the two sides, then the triangle is a **right triangle.** $a^2 + b^2 = c^2$

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an **acute triangle.** $a^2 + b^2 > c^2$

If the square of the length of the longest side of a triangle is less than the sum of the other two side, then the triangle is an **obtuse angle.** $a^2 + b^2 < c$

- Similar right triangles:


THEOREM *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

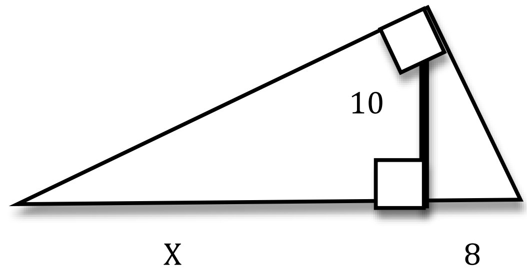
Proof: below; Ex. 35, p. 456

Altitude: $\frac{\text{altitude}}{\text{hypotenuse piece}} = \frac{\text{hypotenuse Piece}}{\text{altitude}}$

$$\frac{10}{x} = \frac{8}{10}$$

$$100 = 8x$$

$$x = 12.5$$

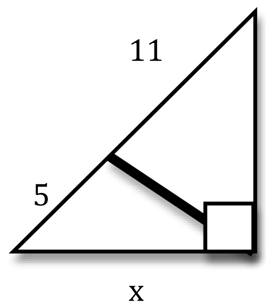


Leg $\frac{\text{short leg}}{\text{Closest \#}} = \frac{\text{hypotenuse}}{\text{short leg}}$

$$\frac{x}{5} = \frac{16}{x}$$

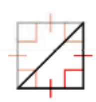
$$x^2 = 80$$

$$x = 4\sqrt{5}$$



- Special right triangles (45-45-90 and 30-60-90)

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.



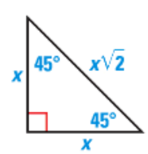
THEOREM *For Your Notebook*

THEOREM 7.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$

Proof: Ex. 30, p. 463



A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

THEOREM *For Your Notebook*

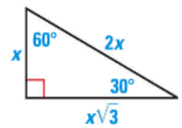
THEOREM 7.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

Proof: Ex. 32, p. 463



- <u>Trigonometric ratios (SOH CAH TOA)</u>

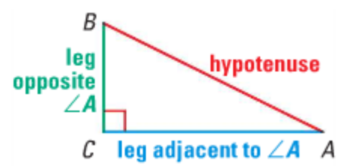
KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

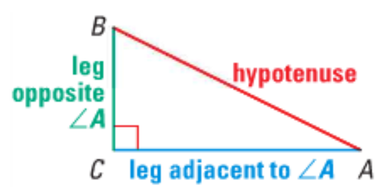


KEY CONCEPT *For Your Notebook*

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

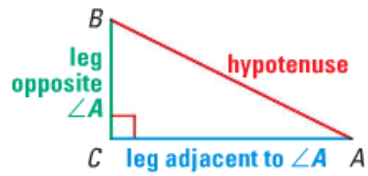


- <u>Solving right triangles (trig inverses)</u>

KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written sin A and cos A) are defined as follows:



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

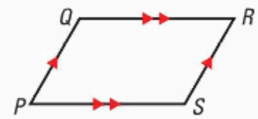
Chapter 8 (All sections)

- <u>Angle Measures in polygons</u>
  - **Polygon interior angles theorem:** (n-2) x 180
  - **Polygon exterior angles theorem:** $\frac{360}{n}$

- <u>Properties of parallelograms</u>

1. Both pairs of sides congruent
2. Both pairs of sides parallel
3. Both pairs of angles congruent
4. One angle supplementary to both consecutive angles
5. Diagonals bisect each other
6. One pair of sides both parallel and congruent

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. The term "parallelogram *PQRS*" can be written as ▱*PQRS*. In ▱*PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.



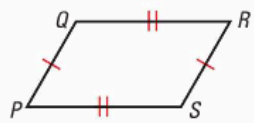
THEOREMS — *For Your Notebook*

THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If *PQRS* is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Proof: p. 516

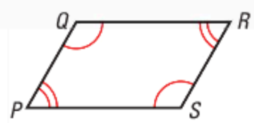


THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof: Ex. 42, p. 520



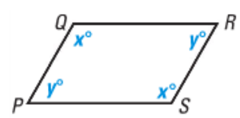
THEOREM — *For Your Notebook*

THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If *PQRS* is a parallelogram, then $x° + y° = 180°$.

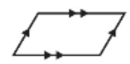
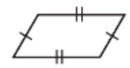
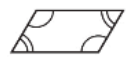
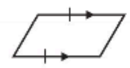
Proof: Ex. 43, p. 520



- <u>Show quadrilaterals are parallelogram</u>

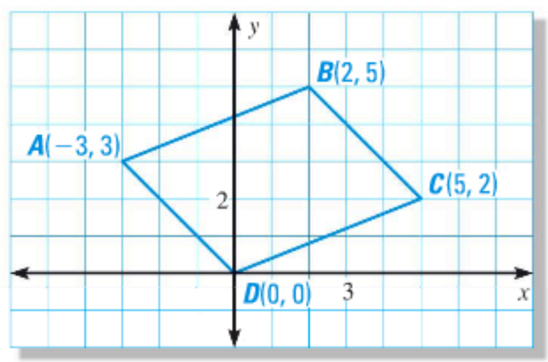
CONCEPT SUMMARY *For Your Notebook*

Ways to Prove a Quadrilateral is a Parallelogram

1. Show both pairs of opposite sides are parallel. *(DEFINITION)* 
2. Show both pairs of opposite sides are congruent. *(THEOREM 8.7)* 
3. Show both pairs of opposite angles are congruent. *(THEOREM 8.8)* 
4. Show one pair of opposite sides are congruent and parallel. *(THEOREM 8.9)* 
5. Show the diagonals bisect each other. *(THEOREM 8.10)* 

EXAMPLE 4 Use coordinate geometry

Show that quadrilateral *ABCD* is a parallelogram.



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Solution

One way is to show that a pair of sides are congruent and parallel. Then apply Theorem 8.9.

First use the Distance Formula to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

$$AB = \sqrt{[2-(-3)]^2 + (5-3)^2} = \sqrt{29} \qquad CD = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{29}$$

Because $AB = CD = \sqrt{29}$, $\overline{AB} \cong \overline{CD}$.

Then use the slope formula to show that $\overline{AB} \parallel \overline{CD}$.

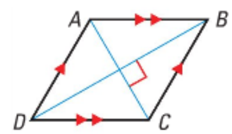
$$\text{Slope of } \overline{AB} = \frac{5-(3)}{2-(-3)} = \frac{2}{5} \qquad \text{Slope of } \overline{CD} = \frac{2-0}{5-0} = \frac{2}{5}$$

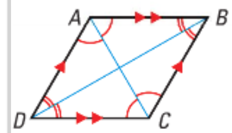
Because $\overline{AB}$ and $\overline{CD}$ have the same slope, they are parallel.

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel. So, *ABCD* is a parallelogram by Theorem 8.9.

- <u>Properties of: rhombuses, rectangles, squares, trapezoids, and kites</u>

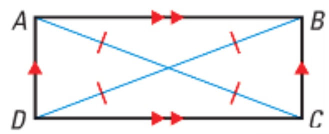
Rhombus:





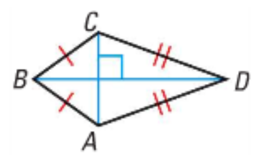
1. All properties of parallelogram
2. All sides congruent
3. Diagonals bisect the opposite angles
4. Diagonals are perpendicular to eachother

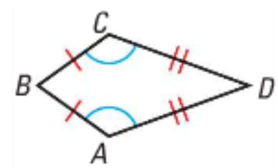
Rectangles:



1. All properties of a parallelogram
2. All angles 90^o
3. Diagonals are congruent

Kites:

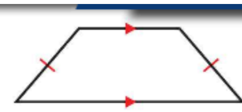




1. Only one pair opposite angles are congruent
2. Diagonals are perpendicular
3. Consecutive sides are congruent
4. NOT a parallelogram

Trapezoids: one pair parallel sides

ISOSCELES TRAPEZOIDS If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



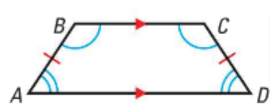
isosceles trapezoid

THEOREMS *For Your Notebook*

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

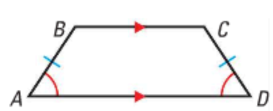


Proof: Ex. 37, p. 548

THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

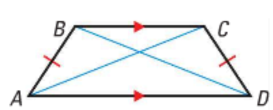


Proof: Ex. 38, p. 548

THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

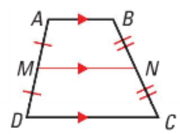


Proof: Exs. 39 and 43, p. 549

THEOREM *For Your Notebook*

THEOREM 8.17 Midsegment Theorem for Trapezoids

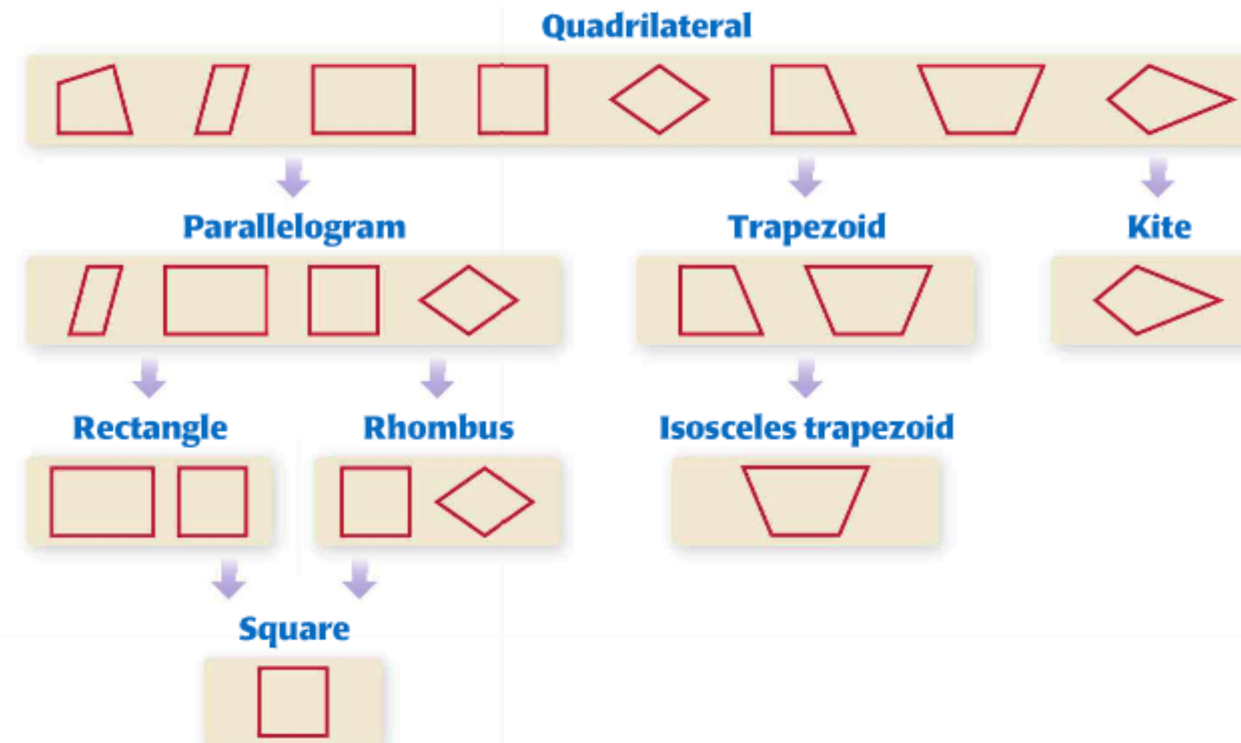
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

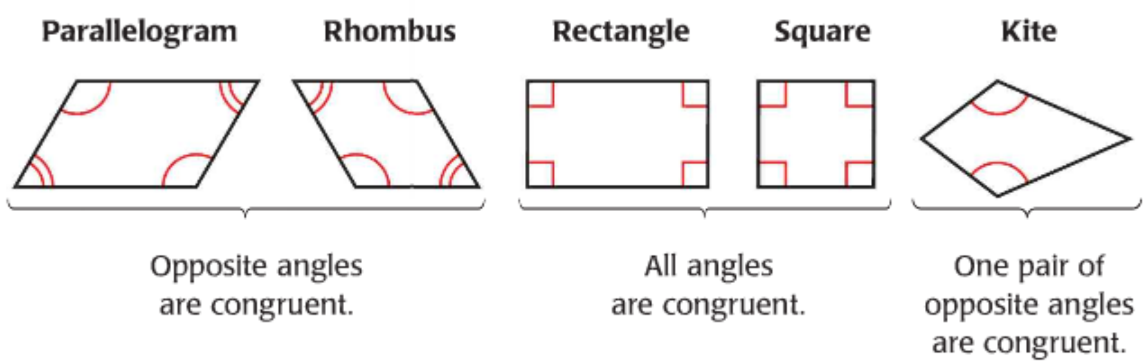


If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Justification: Ex. 40, p. 549
Proof: p. 937

- <u>Identify quadrilaterals in coordinate planes</u>





Chapter 9 (All sections except section 9.2)

- Translations of figures and vectors
  - (x,y)→ (x+a, y+b) **coordinate notation-** means the preimage is translated *a* units horizontally and *b* units vertically.
  - **Vectors:** Have both:
    1. Direction
    2. Magnitude (or length)

*Vectors are often written in **component form** is <x,y>
*The **magnitude, or length,** of a vector can be found by using the Pythagorean Theorem.

- Reflections (lines of symmetry)

KEY CONCEPT *For Your Notebook*

Coordinate Rules for Reflections

- If (a, b) is reflected in the x-axis, its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, its image is the point $(-b, -a)$.

Rotations (angles of rotation)

Coordinate Rules for Rotations

In general, we can state the following coordinate rules for (counterclockwise) rotations about the origin:

For a rotation of 90°: (x, y) --> $(-y, x)$

For a rotation of 180°: (x, y) --> $(-x, -y)$

- For a rotation of 270°: (x, y) --> $(y, -x)$

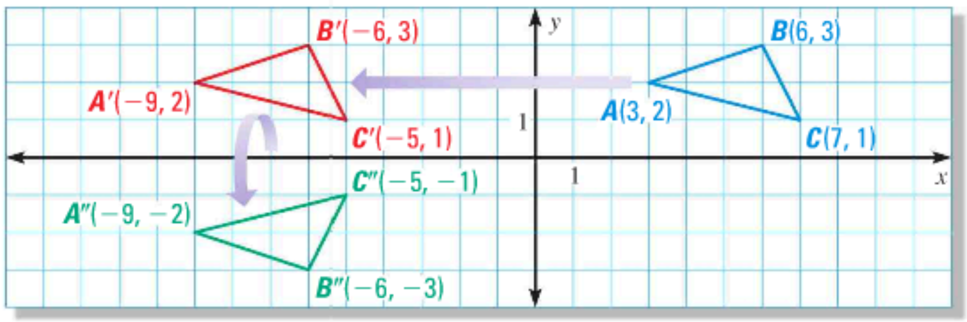
- <u>Glide reflections and compositions</u>

The vertices of $\triangle ABC$ are $A(3, 2)$, $B(6, 3)$, and $C(7, 1)$. Find the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \to (x - 12, y)$
Reflection: in the x-axis

Solution

Begin by graphing $\triangle ABC$. Then graph $\triangle A'B'C'$ after a translation 12 units left. Finally, graph $\triangle A''B''C''$ after a reflection in the x-axis.



EXAMPLE 2 Find the image of a composition

The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(2, -6)$. Graph the image of $\overline{RS}$ after the composition.

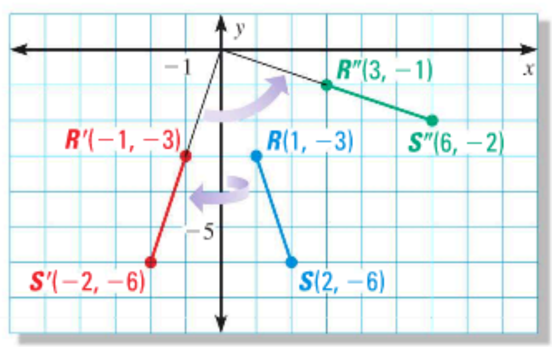
Reflection: in the y-axis
Rotation: $90°$ about the origin

Solution

STEP 1 **Graph** $\overline{RS}$.

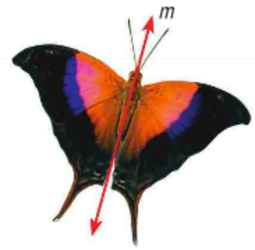
STEP 2 **Reflect** $\overline{RS}$ in the y-axis. $\overline{R'S'}$ has endpoints $R'(-1, -3)$ and $S'(-2, -6)$.

STEP 3 **Rotate** $\overline{R'S'}$ $90°$ about the origin. $\overline{R''S''}$ has endpoints $R''(3, -1)$ and $S''(6, -2)$.



- <u>Identify symmetry</u>

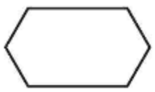
A figure in the plane has **line symmetry** if the figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry**, such as line *m* at the right. A figure can have more than one line of symmetry.



EXAMPLE 1 Identify lines of symmetry

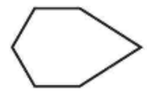
How many lines of symmetry does the hexagon have?

a.



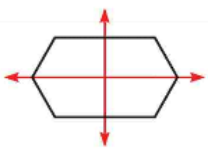
b.

c.

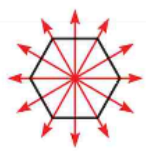


Solution

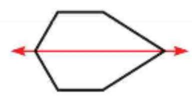
a. Two lines of symmetry



b. Six lines of symmetry



c. One line of symmetry

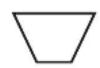


Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a. Parallelogram

b. Regular octagon

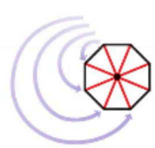
c. Trapezoid



Solution

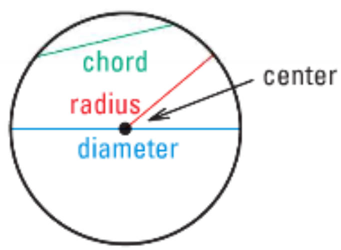
a. The parallelogram has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the parallelogram onto itself.

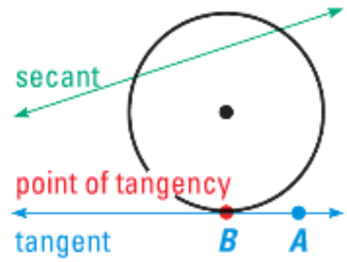
b. The regular octagon has rotational symmetry. The center is the intersection of the diagonals. Rotations of 45°, 90°, 135°, or 180° about the center all map the octagon onto itself.



c. The trapezoid does not have rotational symmetry because no rotation of 180° or less maps the trapezoid onto itself.

Chapter 10 (All sections except section 10.5)





- <u>Circle Vocabulary :</u>

A **circle** is the set of all points in a plane that are equidistant from a given point, called the **center** of the circle.

The distance from the center to a point on the circle is the **radius** of the circle.

Two circles are **congruent** if they have the same radius.

The distance across the circle, through the center is the **diameter** of the circle.

A **radius** is a segment whose endpoints are the center of the circle and a point on the circle.

A **chord** is a segment whose endpoints are points on the circle.

A **diameter** is a chord that passes through the center of the circle.

A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle and a point on the circle.

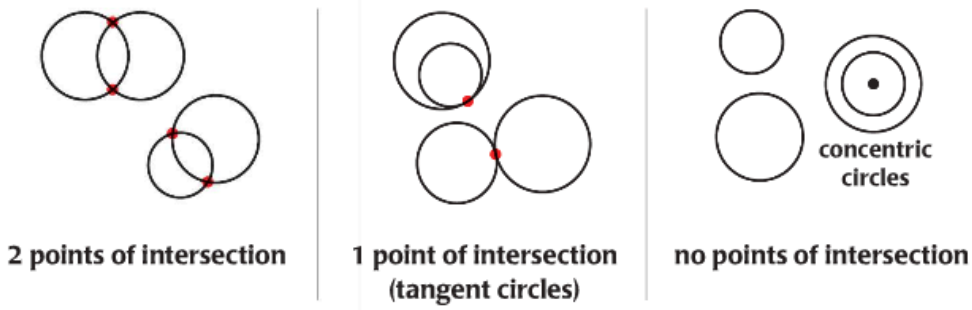
If a line is tangent to a circle, then it is perpendicular to a radius drawn to the point of tangency.

In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.

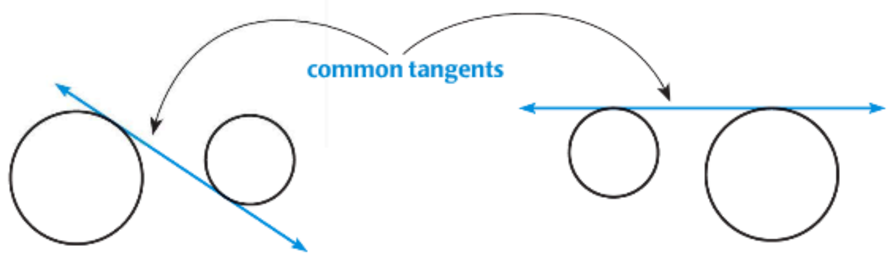
If two segments from the exterior point are tangent to a circle, then they are congruent.

- Tangents and their properties

COPLANAR CIRCLES Two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called *tangent circles*. Coplanar circles that have a common center are called *concentric*.



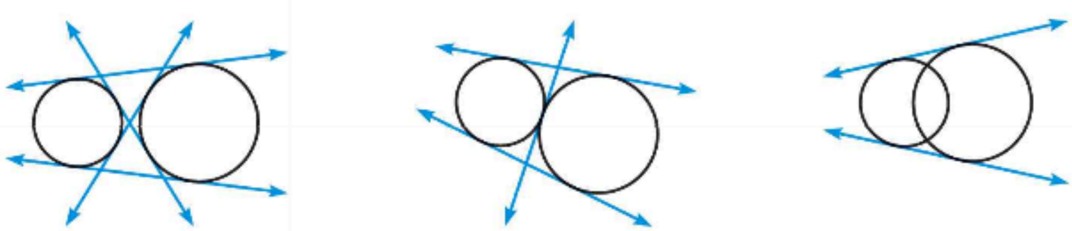
COMMON TANGENTS A line, ray, or segment that is tangent to two coplanar circles is called a *common tangent*.



Tell how many common tangents the circles have and draw them.

Solution

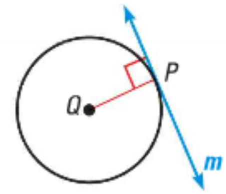
a. 4 common tangents **b.** 3 common tangents **c.** 2 common tangents



THEOREM — *For Your Notebook*

THEOREM 10.1

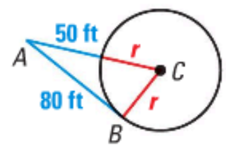
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.



Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof: Exs. 39–40, p. 658

In the diagram, B is a point of tangency. Find the radius r of $\odot C$.



Solution

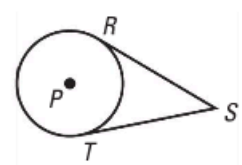
You know from Theorem 10.1 that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem.

$AC^2 = BC^2 + AB^2$	Pythagorean Theorem
$(r + 50)^2 = r^2 + 80^2$	Substitute.
$r^2 + 100r + 2500 = r^2 + 6400$	Multiply.
$100r = 3900$	Subtract from each side.
$r = 39$ ft	Divide each side by 100.

THEOREM — *For Your Notebook*

THEOREM 10.2

Tangent segments from a common external point are congruent.



If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

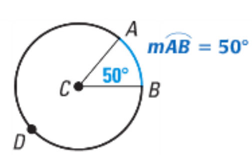
Proof: Ex. 41, p. 658

- <u>Arcs and inscribed angles :</u>

KEY CONCEPT — *For Your Notebook*

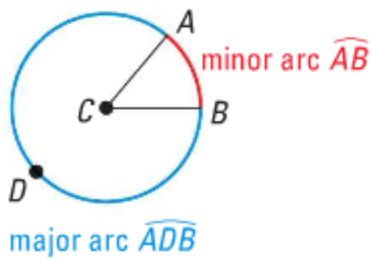
Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."



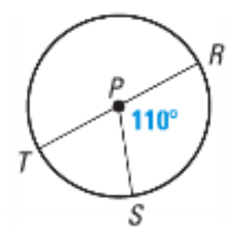
$m\overset{\frown}{ADB} = 360° - 50° = 310°$

The measure of the entire circle is 360°. The **measure of a major arc** is the difference between 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



- Chords and their properties
- Segment relationships
- Equations of circles

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.



a. $\widehat{RS}$ **b.** $\widehat{RTS}$ **c.** $\widehat{RST}$

Solution

a. $\widehat{RS}$ is a minor arc, so $m\widehat{RS} = m\angle RPS = 110°$.

b. $\widehat{RTS}$ is a major arc, so $m\widehat{RTS} = 360° - 110° = 250°$.

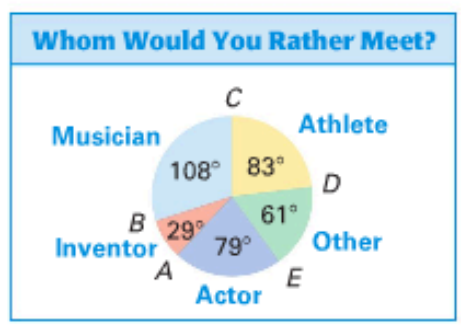
c. $\overline{RT}$ is a diameter, so $\widehat{RST}$ is a semicircle, and $m\widehat{RST} = 180°$.

EXAMPLE 2 Find measures of arcs

SURVEY A recent survey asked teenagers if they would rather meet a famous musician, athlete, actor, inventor, or other person. The results are shown in the circle graph. Find the indicated arc measures.

a. $m\widehat{AC}$ **b.** $m\widehat{ACD}$

c. $m\widehat{ADC}$ **d.** $m\widehat{EBD}$



Solution

a. $m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$
$= 29° + 108°$
$= 137°$

b. $m\widehat{ACD} = m\widehat{AC} + m\widehat{CD}$
$= 137° + 83°$
$= 220°$

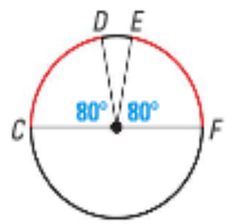
c. $m\widehat{ADC} = 360° - m\widehat{AC}$
$= 360° - 137°$
$= 223°$

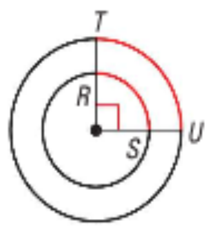
d. $m\widehat{EBD} = 360° - m\widehat{ED}$
$= 360° - 61°$
$= 299°$

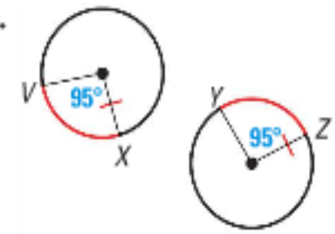
CONGRUENT CIRCLES AND ARCS Two circles are **congruent circles** if they have the same radius. Two arcs are **congruent arcs** if they have the same measure and they are arcs of the same circle or of congruent circles. If $\odot C$ is congruent to $\odot D$, then you can write $\odot C \cong \odot D$.

EXAMPLE 3 Identify congruent arcs

Tell whether the red arcs are congruent. Explain why or why not.

a. 

b. 

c. 

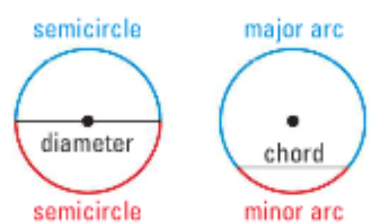
Solution

a. $\widehat{CD} \cong \widehat{EF}$ because they are in the same circle and $m\widehat{CD} = m\widehat{EF}$.

b. $\widehat{RS}$ and $\widehat{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\widehat{VX} \cong \widehat{YZ}$ because they are in congruent circles and $m\widehat{VX} = m\widehat{YZ}$.

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.

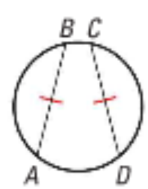


THEOREM *For Your Notebook*

THEOREM 10.3

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

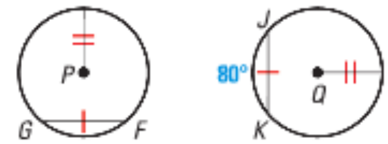
Proof: Exs. 27–28, p. 669



$\widehat{AB} \cong \widehat{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

EXAMPLE 1 Use congruent chords to find an arc measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\widehat{JK} = 80°$. Find $m\widehat{FG}$.

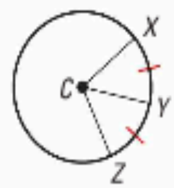


Solution

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\widehat{FG}$ and $\widehat{JK}$ are congruent.

▸ So, $m\widehat{FG} = m\widehat{JK} = 80°$.

BISECTING ARCS If $\widehat{XY} \cong \widehat{YZ}$, then the point Y, and any line, segment, or ray that contains Y, *bisects* $\widehat{XYZ}$.



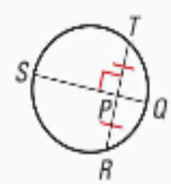
$\overline{CY}$ bisects $\widehat{XYZ}$.

THEOREMS *For Your Notebook*

THEOREM 10.4

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

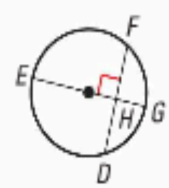


Proof: Ex. 31, p. 670

THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\widehat{GD} \cong \widehat{GF}$.



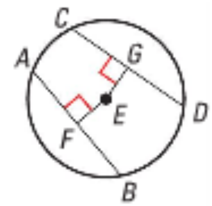
Proof: Ex. 32, p. 670

THEOREM *For Your Notebook*

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

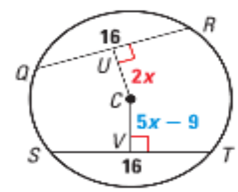
Proof: Ex. 33, p. 670



$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

EXAMPLE 4 Use Theorem 10.6

In the diagram of $\odot C$, $QR = ST = 16$. Find CU.



Solution

Chords $\overline{QR}$ and $\overline{ST}$ are congruent, so by Theorem 10.6 they are equidisant from C. Therefore, $CU = CV$.

$CU = CV$ — Use Theorem 10.6.

$2x = 5x - 9$ — Substitute.

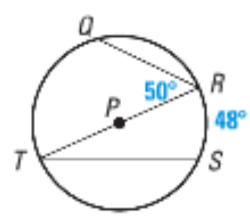
$x = 3$ — Solve for x.

▶ So, $CU = 2x = 2(3) = 6$.

EXAMPLE 1 Use inscribed angles

Find the indicated measure in $\odot P$.

a. $m\angle T$ **b.** $m\widehat{QR}$

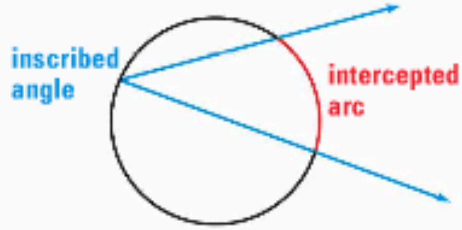


Solution

a. $m\angle T = \frac{1}{2}m\widehat{RS} = \frac{1}{2}(48°) = 24°$

b. $m\widehat{TQ} = 2m\angle R = 2 \cdot 50° = 100°$. Because $\widehat{TQR}$ is a semicircle, $m\widehat{QR} = 180° - m\widehat{TQ} = 180° - 100° = 80°$. So, $m\widehat{QR} = 80°$.

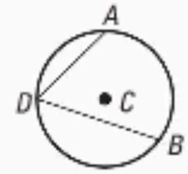
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.



THEOREM — *For Your Notebook*

THEOREM 10.7 Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one half the measure of its intercepted arc.

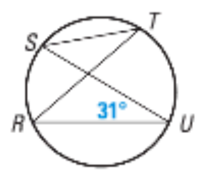


$m\angle ADB = \frac{1}{2}m\widehat{AB}$

Proof: Exs. 31–33, p. 678

EXAMPLE 2 Find the measure of an intercepted arc

Find $m\widehat{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



Solution

From Theorem 10.7, you know that $m\widehat{RS} = 2m\angle RUS = 2(31°) = 62°$.

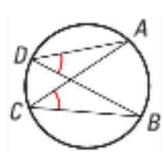
Also, $m\angle STR = \frac{1}{2}m\widehat{RS} = \frac{1}{2}(62°) = 31°$. So, $\angle STR \cong \angle RUS$.

INTERCEPTING THE SAME ARC Example 2 suggests Theorem 10.8.

THEOREM — *For Your Notebook*

THEOREM 10.8

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong \angle ACB$

Proof: Ex. 34, p. 678

EXAMPLE 3 Standardized Test Practice

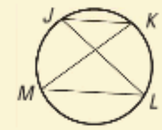
Name two pairs of congruent angles in the figure.

Ⓐ $\angle JKM \cong \angle KJL$, $\angle JLM \cong \angle KML$

Ⓑ $\angle JLM \cong \angle KJL$, $\angle JKM \cong \angle KML$

Ⓒ $\angle JKM \cong \angle JLM$, $\angle KJL \cong \angle KML$

Ⓓ $\angle JLM \cong \angle KJL$, $\angle JLM \cong \angle JKM$



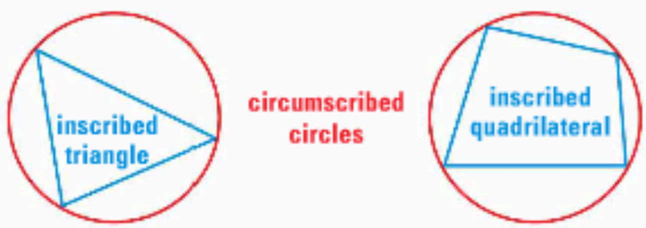
Solution

ELIMINATE CHOICES You can eliminate choices A and B, because they do not include the pair $\angle JKM \cong \angle JLM$.

Notice that $\angle JKM$ and $\angle JLM$ intercept the same arc, and so $\angle JKM \cong \angle JLM$ by Theorem 10.8. Also, $\angle KJL$ and $\angle KML$ intercept the same arc, so they must also be congruent. Only choice C contains both pairs of angles.

▶ So, by Theorem 10.8, the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

POLYGONS A polygon is an **inscribed polygon** if all of its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

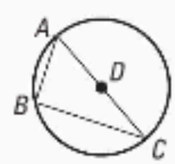


THEOREM *For Your Notebook*

THEOREM 10.9

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

Proof: Ex. 35, p. 678

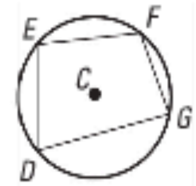


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

THEOREM 10.10

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

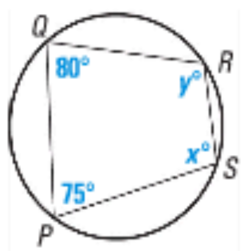


Proof: Ex. 30, p. 678; p. 938

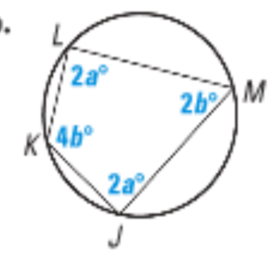
EXAMPLE 5 Use Theorem 10.10

Find the value of each variable.

a.



b.



Solution

a. $PQRS$ is inscribed in a circle, so opposite angles are supplementary.

$m\angle P + m\angle R = 180°$
$75° + y° = 180°$
$y = 105$

$m\angle Q + m\angle S = 180°$
$80° + x° = 180°$
$x = 100$

b. $JKLM$ is inscribed in a circle, so opposite angles are supplementary.

$m\angle J + m\angle L = 180°$
$2a° + 2a° = 180°$
$4a = 180$
$a = 45$

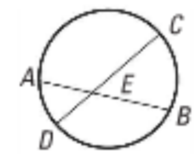
$m\angle K + m\angle M = 180°$
$4b° + 2b° = 180°$
$6b = 180$
$b = 30$

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

THEOREM — *For Your Notebook*

THEOREM 10.14 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

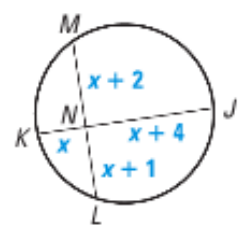


$EA \cdot EB = EC \cdot ED$

Proof: Ex. 21, p. 694

EXAMPLE 1 Find lengths using Theorem 10.14

ALGEBRA **Find *ML* and *JK*.**



Solution

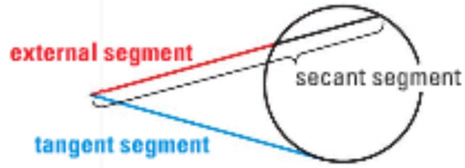
$NK \cdot NJ = NL \cdot NM$	Use Theorem 10.14.
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	Substitute.
$x^2 + 4x = x^2 + 3x + 2$	Simplify.
$4x = 3x + 2$	Subtract x^2 from each side.
$x = 2$	Solve for x.

Find *ML* and *JK* by substitution.

$$ML = (x + 2) + (x + 1) = 2 + 2 + 2 + 1 = 7$$

$$JK = x + (x + 4) = 2 + 2 + 4 = 8$$

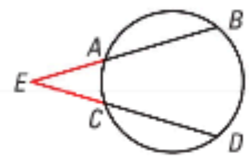
TANGENTS AND SECANTS A **secant segment** is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



THEOREM *For Your Notebook*

THEOREM 10.15 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot EB = EC \cdot ED$

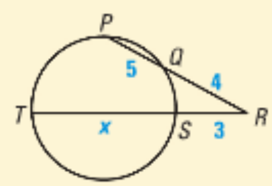
Proof: Ex. 25, p. 694

★ EXAMPLE 2 Standardized Test Practice

What is the value of x?

Ⓐ 6 Ⓑ $6\frac{2}{3}$

Ⓒ 8 Ⓓ 9



Solution

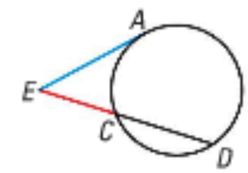
$RQ \cdot RP = RS \cdot RT$	**Use Theorem 10.15.**
$4 \cdot (5 + 4) = 3 \cdot (x + 3)$	**Substitute.**
$36 = 3x + 9$	**Simplify.**
$9 = x$	**Solve for *x*.**

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

THEOREM *For Your Notebook*

THEOREM 10.16 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

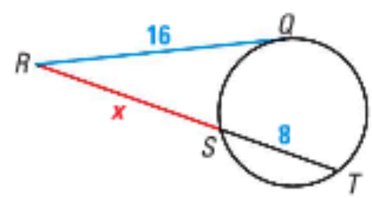


$EA^2 = EC \cdot ED$

Proof: Ex. 26, p. 694

EXAMPLE 3 Find lengths using Theorem 10.16

Use the figure at the right to find *RS*.



Solution

$RQ^2 = RS \cdot RT$ — **Use Theorem 10.16.**

$16^2 = x \cdot (x + 8)$ — **Substitute.**

$256 = x^2 + 8x$ — **Simplify.**

$0 = x^2 + 8x - 256$ — **Write in standard form.**

$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$ — **Use quadratic formula.**

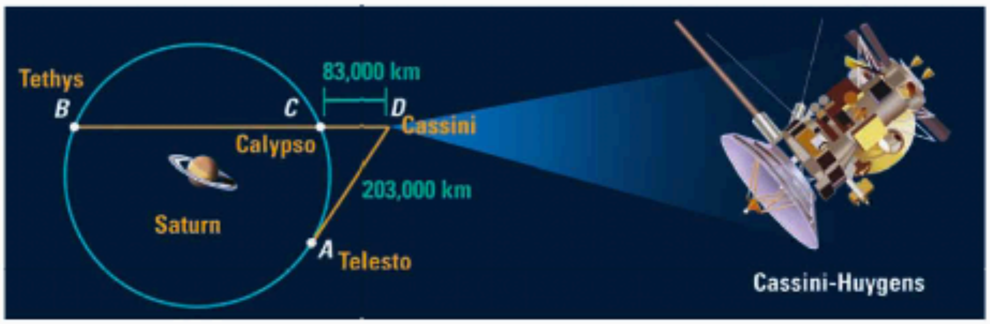
$x = -4 \pm 4\sqrt{17}$ — **Simplify.**

Use the positive solution, because lengths cannot be negative.

▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

EXAMPLE 4 Solve a real-world problem

SCIENCE Tethys, Calypso, and Telesto are three of Saturn's moons. Each has a nearly circular orbit 295,000 kilometers in radius. The Cassini-Huygens spacecraft entered Saturn's orbit in July 2004. Telesto is on a point of tangency. Find the distance DB from Cassini to Tethys.



Solution

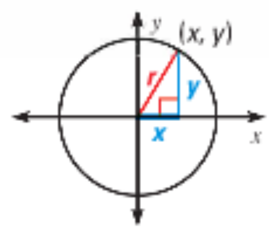
$DC \cdot DB = AD^2$	Use Theorem 10.16.
$83{,}000 \cdot DB \approx 203{,}000^2$	Substitute.
$DB \approx 496{,}494$	Solve for DB.

▶ Cassini is about 496,494 kilometers from Tethys.

Let (x, y) represent any point on a circle with center at the origin and radius r. By the Pythagorean Theorem,

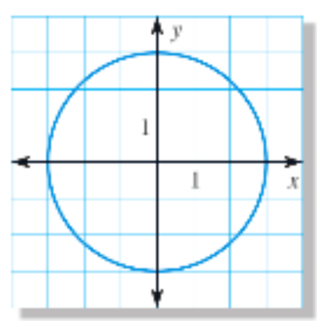
$$x^2 + y^2 = r^2.$$

This is the equation of a circle with radius r and center at the origin.



EXAMPLE 1 Write an equation of a circle

Write the equation of the circle shown.



Solution

The radius is 3 and the center is at the origin.

$x^2 + y^2 = r^2$	Equation of circle
$x^2 + y^2 = 3^2$	Substitute.
$x^2 + y^2 = 9$	Simplify.

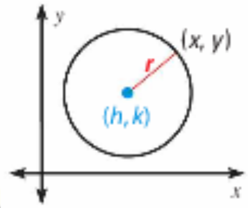
▶ The equation of the circle is $x^2 + y^2 = 9$.

CIRCLES CENTERED AT (h, k) You can write the equation of *any* circle if you know its radius and the coordinates of its center.

Suppose a circle has radius r and center (h, k). Let (x, y) be a point on the circle. The distance between (x, y) and (h, k) is r, so by the Distance Formula

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

Square both sides to find the **standard equation of a circle**.



KEY CONCEPT — *For Your Notebook*

Standard Equation of a Circle

The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLE 2 Write the standard equation of a circle

Write the standard equation of a circle with center (0, −9) and radius 4.2.

Solution

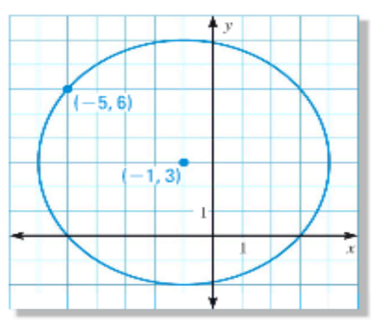
$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$(x - 0)^2 + (y - (-9))^2 = 4.2^2$ — Substitute.

$x^2 + (y + 9)^2 = 17.64$ — Simplify.

EXAMPLE 3 Write the standard equation of a circle

The point (−5, 6) is on a circle with center (−1, 3). Write the standard equation of the circle.



Solution

To write the standard equation, you need to know the values of h, k, and r. To find r, find the distance between the center and the point $(-5, 6)$ on the circle.

$r = \sqrt{[-5 - (-1)]^2 + (6 - 3)^2}$ — Distance Formula

$= \sqrt{(-4)^2 + 3^2}$ — Simplify.

$= 5$ — Simplify.

Substitute $(h, k) = (-1, 3)$ and $r = 5$ into the standard equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$[x - (-1)]^2 + (y - 3)^2 = 5^2$ — Substitute.

$(x + 1)^2 + (y - 3)^2 = 25$ — Simplify.

▶ The standard equation of the circle is $(x + 1)^2 + (y - 3)^2 = 25$.

EXAMPLE 4 Graph a circle

USE EQUATIONS
If you know the equation of a circle, you can graph the circle by identifying its center and radius.

The equation of a circle is $(x - 4)^2 + (y + 2)^2 = 36$. Graph the circle.

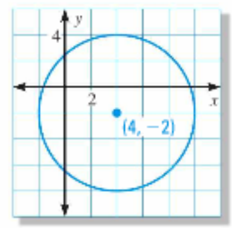
Solution

Rewrite the equation to find the center and radius.

$$(x - 4)^2 + (y + 2)^2 = 36$$

$$(x - 4)^2 + [y - (-2)]^2 = 6^2$$

The center is $(4, -2)$ and the radius is 6. Use a compass to graph the circle.

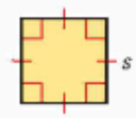


Chapter 11 (All sections)

POSTULATES *For Your Notebook*

POSTULATE 24 Area of a Square Postulate

The area of a square is the square of the length of its side.



POSTULATE 25 Area Congruence Postulate

If two polygons are congruent, then they have the same area.

POSTULATE 26 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

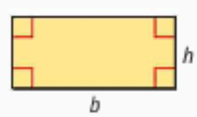
RECTANGLES A rectangle that is b units by h units can be split into $b \cdot h$ unit squares, so the area formula for a rectangle follows from Postulates 24 and 26.

THEOREM *For Your Notebook*

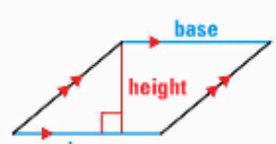
THEOREM 11.1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

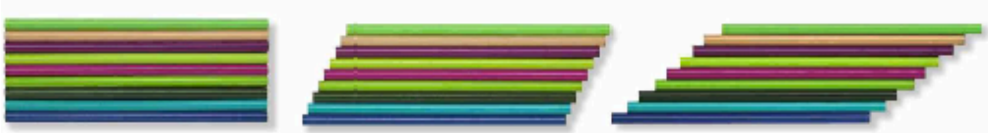
Justification: Ex. 46, p. 726



PARALLELOGRAMS Either pair of parallel sides can be used as the **bases** of a parallelogram. The **height** is the perpendicular distance between these bases.



If you transform a rectangle to form other parallelograms with the same base and height, the area stays the same.

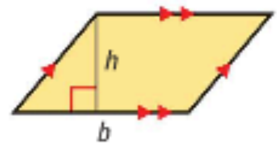


THEOREMS — *For Your Notebook*

THEOREM 11.2 Area of a Parallelogram

The area of a parallelogram is the product of a base and its corresponding height.

Justification: Ex. 42, p. 725

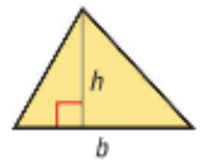


$A = bh$

THEOREM 11.3 Area of a Triangle

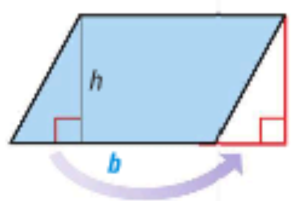
The area of a triangle is one half the product of a base and its corresponding height.

Justification: Ex. 43, p. 726

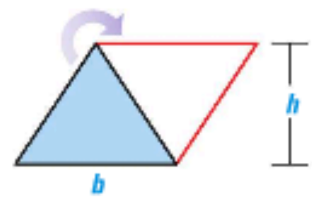


$A = \frac{1}{2}bh$

RELATING AREA FORMULAS As illustrated below, the area formula for a parallelogram is related to the formula for a rectangle, and the area formula for a triangle is related to the formula for a parallelogram. You will write a justification of these relationships in Exercises 42 and 43 on pages 725–726.



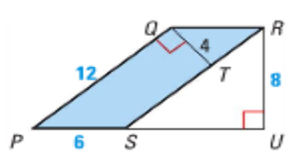
Area of ▱ = Area of Rectangle



Area of △ = $\frac{1}{2}$ • Area of ▱

EXAMPLE 1 **Use a formula to find area**

Find the area of ▱*PQRS*.



Solution

Method 1 Use $\overline{PS}$ as the base.
The base is extended to measure the height *RU*. So, $b = 6$ and $h = 8$.

Area $= bh = 6(8) = 48$ square units

Method 2 Use $\overline{PQ}$ as the base.
Then the height is *QT*. So, $b = 12$ and $h = 4$.

Area $= bh = 12(4) = 48$ square units

EXAMPLE 3 Solve a multi-step problem

PAINTING You need to buy paint so that you can paint the side of a barn. A gallon of paint covers 350 square feet. How many gallons should you buy?

Solution

You can use a right triangle and a rectangle to approximate the area of the side of the barn.

ANOTHER WAY
In Example 3, you have a 45°-45°-90° triangle, so you can also find x by using trigonometry or special right angles.

STEP 1 **Find** the length x of each leg of the triangle.

$26^2 = x^2 + x^2$ — Use Pythagorean Theorem.

$676 = 2x^2$ — Simplify.

$\sqrt{338} = x$ — Solve for the positive value of x.

STEP 2 **Find** the approximate area of the side of the barn.

Area = Area of rectangle + Area of triangle

$= 26(18) + \frac{1}{2} \cdot \left[(\sqrt{338})(\sqrt{338})\right] = 637 \text{ ft}^2$

STEP 3 **Determine** how many gallons of paint you need.

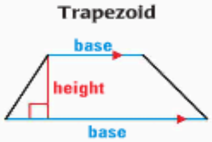
$637 \text{ ft}^2 \cdot \frac{1 \text{ gal}}{350 \text{ ft}^2} \approx 1.82 \text{ gal}$ — Use unit analysis.

▸ Round up so you will have enough paint. You need to buy 2 gallons of paint.

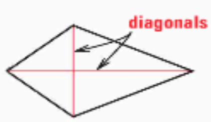
Areas of Trapezoids, Rhombuses, and Kites

The **height of a trapezoid** is the perpendicular distance between its bases.

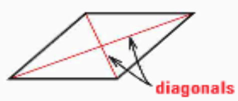
Trapezoid



Kite



Rhombus

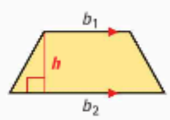


THEOREM *For Your Notebook*

THEOREM 11.4 Area of a Trapezoid

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.

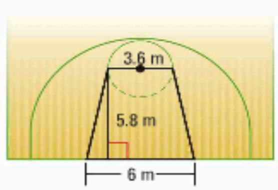
Proof: Ex. 40, p. 736



$A = \frac{1}{2}h(b_1 + b_2)$

EXAMPLE 1 Find the area of a trapezoid

BASKETBALL The free-throw lane on an international basketball court is shaped like a trapezoid. Find the area of the free-throw lane.



Solution

The height of the trapezoid is 5.8 meters. The lengths of the bases are 3.6 meters and 6 meters.

$A = \frac{1}{2}h(b_1 + b_2)$ — Formula for area of a trapezoid

$= \frac{1}{2}(5.8)(3.6 + 6)$ — Substitute 5.8 for h, 3.6 for b_1, and 6 for b_2.

$= 27.84$ — Simplify.

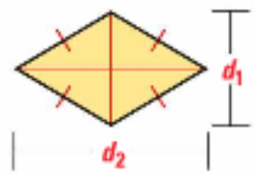
▸ The area of the free-throw lane is about 27.8 square meters.

THEOREMS

For Your Notebook

THEOREM 11.5 Area of a Rhombus

The area of a rhombus is one half the product of the lengths of its diagonals.

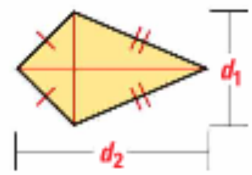


$A = \frac{1}{2}d_1d_2$

Justification: Ex. 39, p. 735

THEOREM 11.6 Area of a Kite

The area of a kite is one half the product of the lengths of its diagonals.

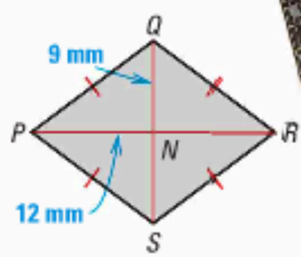


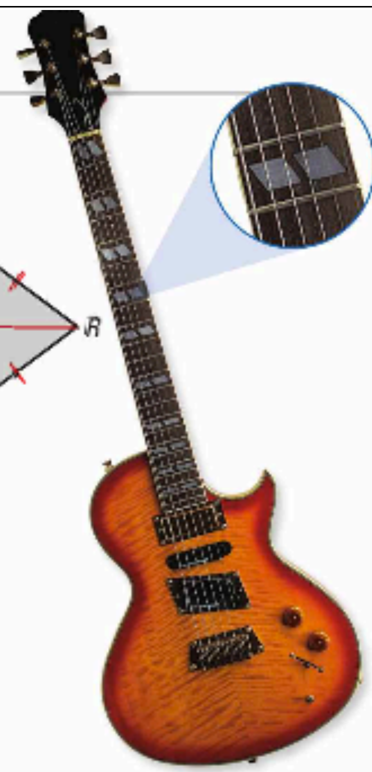
$A = \frac{1}{2}d_1d_2$

Proof: Ex. 41, p. 736

EXAMPLE 2 Find the area of a rhombus

MUSIC Rhombus *PQRS* represents one of the inlays on the guitar in the photo. Find the area of the inlay.





Solution

STEP 1 **Find** the length of each diagonal. The diagonals of a rhombus bisect each other, so $QN = NS$ and $PN = NR$.

$QS = QN + NS = 9 + 9 = 18$ mm

$PR = PN + NR = 12 + 12 = 24$ mm

STEP 2 **Find** the area of the rhombus. Let d_1 represent QS and d_2 represent PR.

$A = \frac{1}{2}d_1d_2$	Formula for area of a rhombus
$= \frac{1}{2}(18)(24)$	Substitute.
$= 216$	Simplify.

▶ The area of the inlay is 216 square millimeters.

Perimeter and area of similar figures:

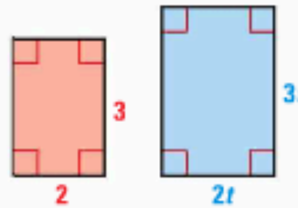
In Chapter 6 you learned that if two polygons are similar, then the ratio of their perimeters, or of any two corresponding lengths, is equal to the ratio of their corresponding side lengths. As shown below, the areas have a different ratio.

Ratio of perimeters

$$\frac{\text{Blue}}{\text{Red}} = \frac{10t}{10} = t$$

Ratio of areas

$$\frac{\text{Blue}}{\text{Red}} = \frac{6t^2}{6} = t^2$$



THEOREM — For Your Notebook

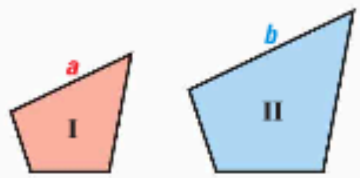
THEOREM 11.7 Areas of Similar Polygons

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Polygon I}}{\text{Side length of Polygon II}} = \frac{a}{b}$$

$$\frac{\text{Area of Polygon I}}{\text{Area of Polygon II}} = \frac{a^2}{b^2}$$

Justification: Ex. 30, p. 742



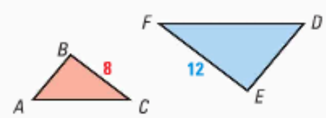
Polygon I ~ Polygon II

EXAMPLE 1 Find ratios of similar polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

a. Ratio (red to blue) of the perimeters

b. Ratio (red to blue) of the areas



Solution

The ratio of the lengths of corresponding sides is $\frac{8}{12} = \frac{2}{3}$, or 2 : 3.

a. By Theorem 6.1 on page 374, the ratio of the perimeters is 2 : 3.

b. By Theorem 11.7 above, the ratio of the areas is $2^2:3^2$, or 4 : 9.

INTERPRET RATIOS
You can also compare the measures with fractions. The perimeter of $\triangle ABC$ is two thirds of the perimeter of $\triangle DEF$. The area of $\triangle ABC$ is four ninths of the area of $\triangle DEF$.

EXAMPLE 3 Use a ratio of areas

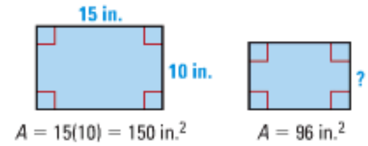
COOKING A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

ANOTHER WAY

For an alternative method for solving the problem in Example 3, turn to page 744 for the **Problem Solving Workshop**.

Solution

First draw a diagram to represent the problem. Label dimensions and areas.



Then use Theorem 11.7. If the area ratio is $a^2 : b^2$, then the length ratio is $a : b$.

$$\frac{\text{Area of smaller pan}}{\text{Area of large pan}} = \frac{96}{150} = \frac{16}{25}$$ **Write ratio of known areas. Then simplify.**

$$\frac{\text{Length in smaller pan}}{\text{Length in large pan}} = \frac{4}{5}$$ **Find square root of area ratio.**

▶ Any length in the smaller pan is $\frac{4}{5}$, or 0.8, of the corresponding length in the large pan. So, the width of the smaller pan is $0.8(10 \text{ inches}) = 8$ inches.

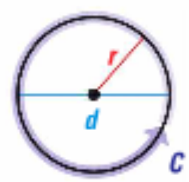
Circumference and arc length:

The **circumference** of a circle is the distance around the circle. For all circles, the ratio of the circumference to the diameter is the same. This ratio is known as π, or *pi*. In Chapter 1, you used 3.14 to approximate the value of π. Throughout this chapter, you should use the π key on a calculator, then round to the hundredths place unless instructed otherwise.

THEOREM — *For Your Notebook*

THEOREM 11.8 Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

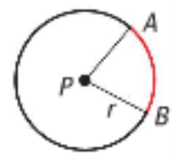
Justification: Ex. 2, p. 769

ARC LENGTH An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

COROLLARY — *For Your Notebook*

ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

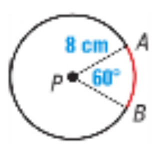


$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

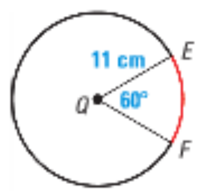
EXAMPLE 3 Find arc lengths

Find the length of each red arc.

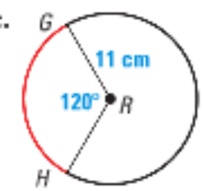
a.



b.



c.



Solution

a. Arc length of $\widehat{AB} = \frac{60^\circ}{360^\circ} \cdot 2\pi(8) \approx 8.38$ centimeters

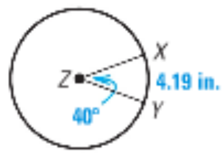
b. Arc length of $\widehat{EF} = \frac{60^\circ}{360^\circ} \cdot 2\pi(11) \approx 11.52$ centimeters

c. Arc length of $\widehat{GH} = \frac{120^\circ}{360^\circ} \cdot 2\pi(11) \approx 23.04$ centimeters

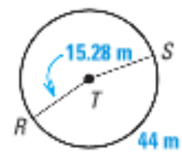
EXAMPLE 4 Use arc lengths to find measures

Find the indicated measure.

a. Circumference C of $\odot Z$



b. $m\widehat{RS}$



Solution

a. $\frac{\text{Arc length of } \widehat{XY}}{C} = \frac{m\widehat{XY}}{360^\circ}$

$\frac{4.19}{C} = \frac{40^\circ}{360^\circ}$

$\frac{4.19}{C} = \frac{1}{9}$

▶ $37.71 = C$

b. $\frac{\text{Arc length of } \widehat{RS}}{2\pi r} = \frac{m\widehat{RS}}{360^\circ}$

$\frac{44}{2\pi(15.28)} = \frac{m\widehat{RS}}{360^\circ}$

$360^\circ \cdot \frac{44}{2\pi(15.28)} = m\widehat{RS}$

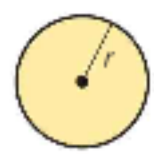
▶ $165^\circ \approx m\widehat{RS}$

Areas of circles and sectors:

THEOREM *For Your Notebook*

THEOREM 11.9 Area of a Circle

The area of a circle is π times the square of the radius.



$A = \pi r^2$

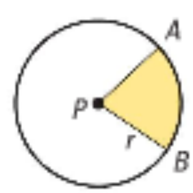
Justification: Ex. 43, p. 761; Ex. 3, p. 769

SECTORS A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\widehat{AB}$. Theorem 11.10 gives a method for finding the area of a sector.

THEOREM *For Your Notebook*

THEOREM 11.10 Area of a Sector

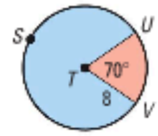
The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.



$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}$, or Area of sector $APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$

EXAMPLE 2 Find areas of sectors

Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 **Find** the measures of the minor and major arcs.

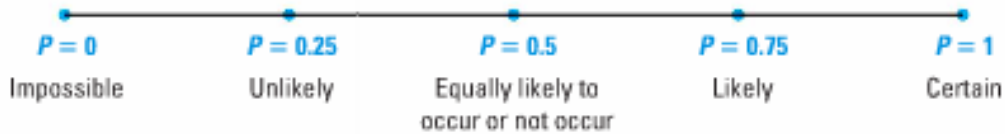
Because $m\angle UTV = 70°$, $m\widehat{UV} = 70°$ and $m\widehat{USV} = 360° - 70° = 290°$.

STEP 2 **Find** the areas of the small and large sectors.

Area of small sector	$= \frac{m\widehat{UV}}{360°} \cdot \pi r^2$	**Write formula for area of a sector.**
	$= \frac{70°}{360°} \cdot \pi \cdot 8^2$	**Substitute.**
	≈ 39.10	**Use a calculator.**
Area of large sector	$= \frac{m\widehat{USV}}{360°} \cdot \pi r^2$	**Write formula for area of a sector.**
	$= \frac{290°}{360°} \cdot \pi \cdot 8^2$	**Substitute.**
	≈ 161.97	**Use a calculator.**

▶ The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.

The **probability** of an event is a measure of the likelihood that the event will occur. It is a number between 0 and 1, inclusive, and can be expressed as a fraction, decimal, or percent. The probability of event A is written as $P(A)$.



In a previous course, you may have found probability by calculating the ratio of the number of favorable outcomes to the total number of possible outcomes. In this lesson, you will find *geometric probabilities.*

A **geometric probability** is a ratio that involves a geometric measure such as length or area.

KEY CONCEPT *For Your Notebook*

Probability and Length

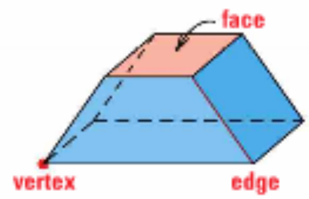
Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.

$$P(K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

Chapter 12 (Only sections 12.1 through 12.5)

Identifying Solids:

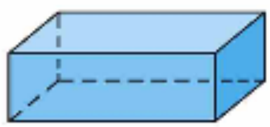
A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



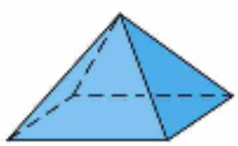
KEY CONCEPT *For Your Notebook*

Types of Solids

Polyhedra

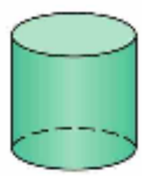


Prism

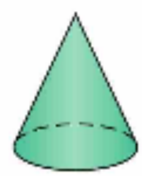


Pyramid

Not Polyhedra



Cylinder

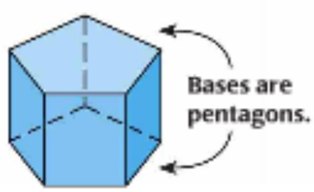


Cone

Sphere

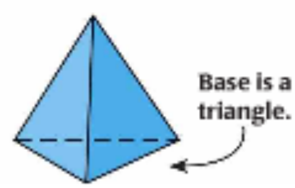
CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.

Pentagonal prism



The two **bases** of a prism are congruent polygons in parallel planes.

Triangular pyramid



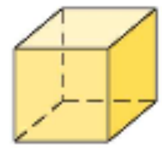
The **base** of a pyramid is a polygon.

EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi'-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

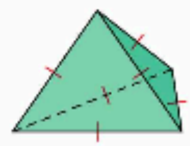
THEOREM 12.1 Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

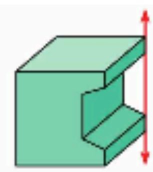


$F = 6, V = 8, E = 12$
$6 + 8 = 12 + 2$

REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.



regular, convex



nonregular, concave

There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.

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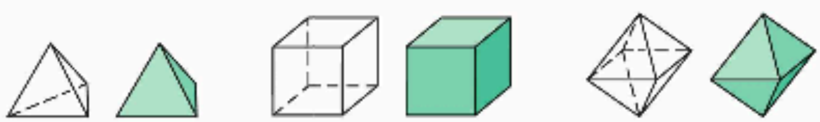
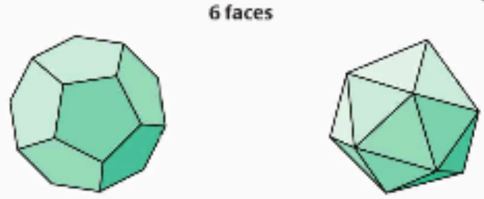


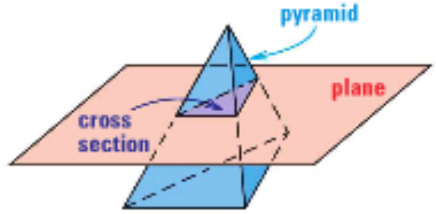
Regular tetrahedron 4 faces | Cube 6 faces | Regular octahedron 8 faces



Regular dodecahedron 12 faces | Regular icosahedron 20 faces

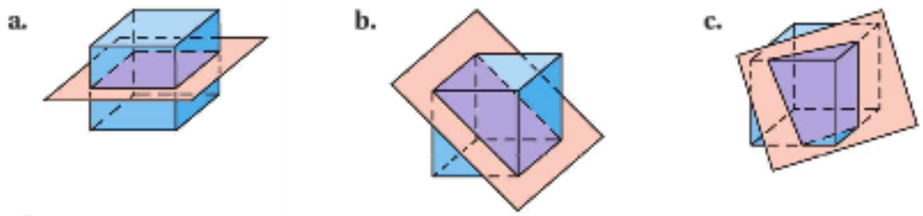
There are only five regular polyhedra because the sum of the measures of the angles that meet at a vertex of a convex polyhedron must be less than 360°. This means that the only possible combinations of regular polygons at a vertex that will form a polyhedron are 3, 4, or 5 triangles, 3 squares, and 3 pentagons.

CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.



Solution

a. The cross section is a square.

b. The cross section is a rectangle.

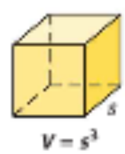
c. The cross section is a trapezoid.

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3).

POSTULATES *For Your Notebook*

POSTULATE 27 Volume of a Cube Postulate

The volume of a cube is the cube of the length of its side.



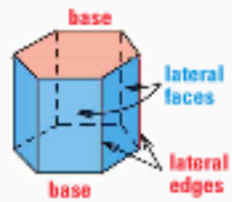
POSTULATE 28 Volume Congruence Postulate

If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 Volume Addition Postulate

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

A **prism** is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

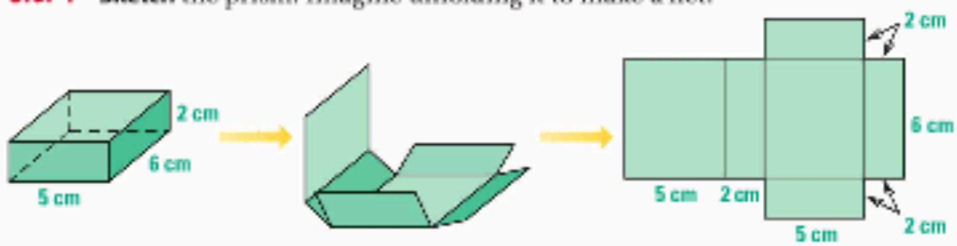
Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. As you saw in the Activity on page 802, the surface area of a prism is equal to the area of its net.

EXAMPLE 1 Use the net of a prism

Find the surface area of a rectangular prism with height 2 centimeters, length 5 centimeters, and width 6 centimeters.

Solution

STEP 1 **Sketch** the prism. Imagine unfolding it to make a net.



STEP 2 **Find** the areas of the rectangles that form the faces of the prism.

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12\text{ cm}^2$
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10\text{ cm}^2$
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30\text{ cm}^2$

STEP 3 **Add** the areas of all the faces to find the surface area.

▶ The surface area of the prism is $S = 2(12) + 2(10) + 2(30) = 104\text{ cm}^2$.

VOLUME FORMULAS The volume of any right prism or right cylinder can be found by multiplying the area of its base by its height.

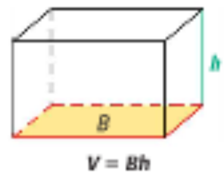
THEOREMS *For Your Notebook*

THEOREM 12.6 Volume of a Prism

The volume V of a prism is

$V = Bh,$

where B is the area of a base and h is the height.

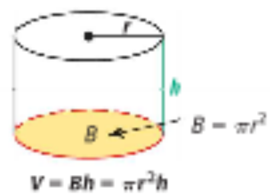


THEOREM 12.7 Volume of a Cylinder

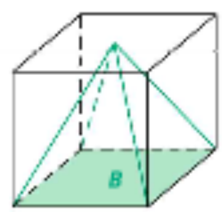
The volume V of a cylinder is

$V = Bh = \pi r^2 h,$

where B is the area of a base, h is the height, and r is the radius of a base.



Recall that the volume of a prism is Bh, where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity on page 828, the volume of a pyramid is one third the volume of a prism.



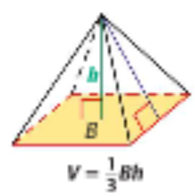
THEOREMS *For Your Notebook*

THEOREM 12.9 Volume of a Pyramid

The volume V of a pyramid is

$V = \frac{1}{3}Bh,$

where B is the area of the base and h is the height.

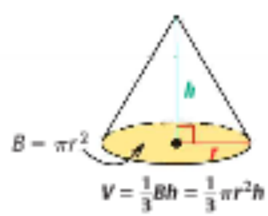


THEOREM 12.10 Volume of a Cone

The volume V of a cone is

$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$

where B is the area of the base, h is the height, and r is the radius of the base.



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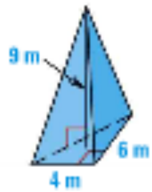
EXAMPLE 1 Find the volume of a solid

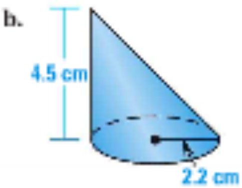
Find the volume of the solid.

a.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$$
$$= 36 \text{ m}^3$$

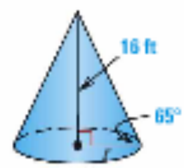
b.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(\pi r^2)h$$
$$= \frac{1}{3}(\pi \cdot 2.2^2)(4.5)$$
$$= 7.26\pi$$
$$\approx 22.81 \text{ cm}^3$$

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



Solution

To find the radius r of the base, use trigonometry.

$\tan 65° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio.**

$\tan 65° = \frac{16}{r}$ **Substitute.**

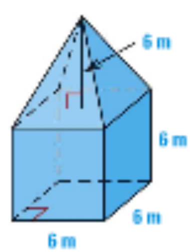
$r = \frac{16}{\tan 65°} \approx 7.46$ **Solve for *r*.**

Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$$

EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.



Solution

Volume of solid = Volume of cube + Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ **Write formulas.**

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ **Substitute.**

$= 216 + 72$ **Simplify.**

$= 288$ **Add.**

▸ The volume of the solid is 288 cubic meters.